WHAT ARE THE DRAG FORCES ON THE LOOP AND HOW DO THEY SLOW ITS FALL?

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This report summarizes a model for the drag forces acting on the front of a loop. These forces decelerate of the loop as it propagates horizontally but they also contribute to keeping it aloft. The drag forces result from both "skin friction" and "form drag" on the loop.

The loop front is assumed to be semi-circular, the upper leg is assumed to travel to the right with velocity $\vec{v}_o$, and the lower leg is assumed to be at rest as shown in the figure below. Thus, the velocity of point "t" at the "top" of the loop is $\vec{v}_o$, while the velocity of the point "b" at the bottom of the loop is zero. The velocity of any point "p" on the loop front will necessarily be smaller that at "t" and it will have both horizontal and vertical components. We must first determine this "velocity field" prior to modeling the skin friction and form drag on the loop front.

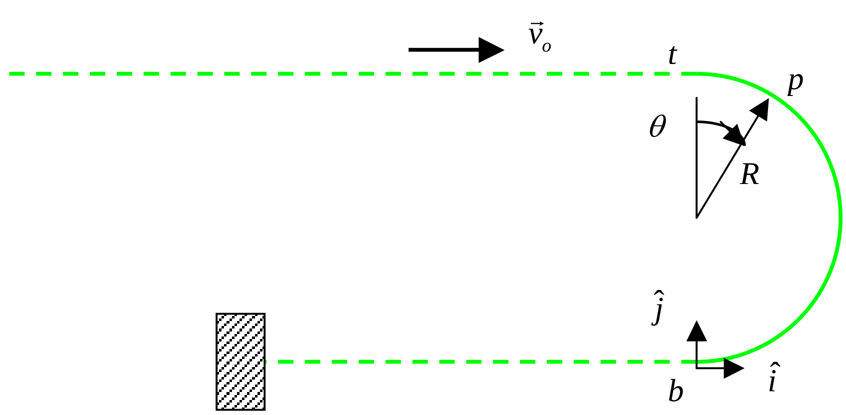

Figure 1. Definition of a propagating loop composed of a semi-circular loop front and upper (traveling) and lower (stationary) segments.

Velocity Field

Let $(\hat{i}, \hat{j}, \hat{k})$ be a frame of reference attached to the loop and at this instant located at the bottom point "*b*". The unit vector $\hat{k}$ points out of the page. The angular velocity of this frame of reference is the angular velocity of the loop and this is given by $\vec{\omega} = -(v_0 / 2R)\hat{k}$ where v_o is the speed of the upper leg. Let θ denote the angle formed between the vertical and the radial line to any point "p" on the loop front. We can now compute the velocity of point "*p*" by using

$$\vec{v}_p = \vec{\omega} \times \vec{r}_{bp}$$

where $\vec{r}_{bp}$ is the position vector locating *p* relative to *b*. *Thus, the velocity at p is perpendicular to* $\vec{r}_{bp}$. Substituting into this the expressions for $\vec{\omega}$ and $\vec{r}_{bp}$ yields

$$\vec{v}_p = -(v_o / 2R)\hat{k} \times R[\sin\theta\hat{i} + (1+\cos\theta)\hat{j}]$$

and this reduces to

$$\vec{v}_p = -\frac{v_o}{2}\sin\theta\hat{j} + \frac{v_0}{2}[1+\cos\theta]\hat{i} \qquad (1.1)$$

Notice that the velocity at the top of the loop ($\theta = 0$) is $v_o\hat{i}$ and that the velocity at the bottom of the loop ($\theta = \pi$) is zero as required. The velocity at the "nose" of the loop ($\theta = \pi/2$) has magnitude (speed) $\frac{\sqrt{2}}{2}v_o$ (about 70% of v_o) and it is directed a 45^o angle below the horizontal. In general, the velocity at any point *p* is directed at a right angle from the position vector $\vec{r}_{bp}$ as shown in the sketch below.

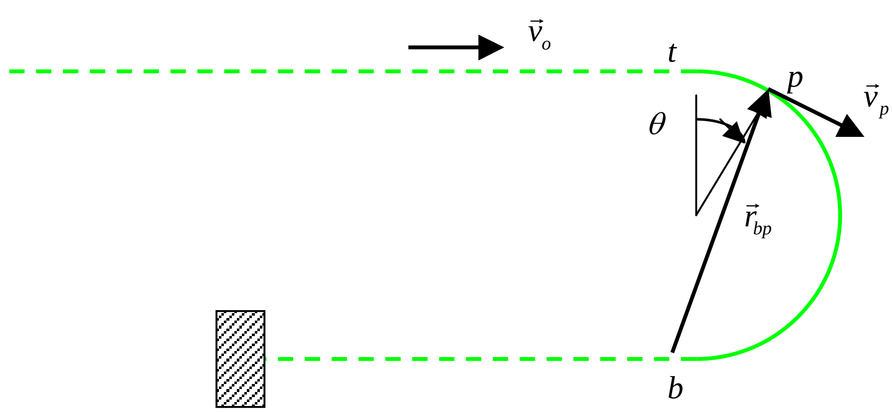

Figure 2. The velocity at point *p* is at right angles to the position vector from the bottom *b* to point *p*.

Notice that the speed $v_p = |\vec{v}_p|$ (the magnitude of the velocity vector) "decays" from a maximum value at the top of the loop to zero at the bottom. Moreover, the direction of the velocity vector changes from being horizontal (to the right) at the top to very nearly vertical (downwards) near the bottom. The speed at any point p is given by

$$v_p = |\vec{v}_p| = \frac{\sqrt{2}}{2} v_o \sqrt{1+\cos\theta}$$

Below is a plot that shows how the speed decays around the loop in terms of the percentage of the speed at the top.

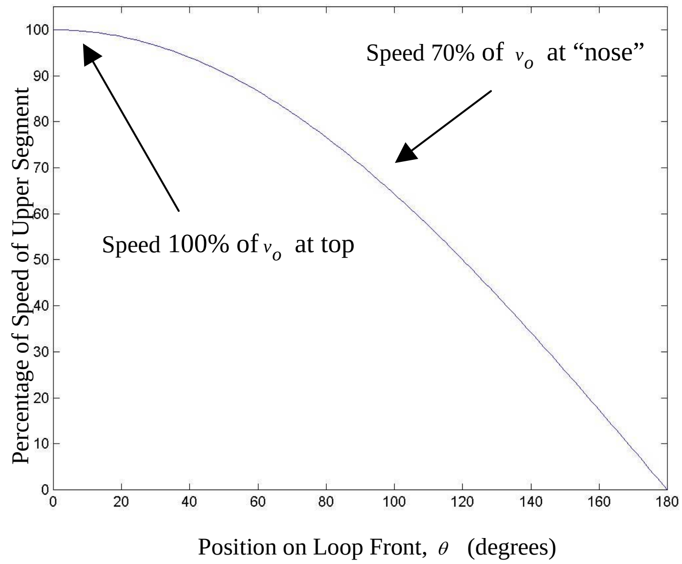

Figure 3. The speed decays from a maximum at the top of the loop ($\theta = 0^o$) to zero at the bottom of the loop ($\theta = 180^o$). The speed at the "nose" of the loop ($\theta = 90^o$) is still 70% of that at the top.

With the above understanding of the velocity field, we can now formulate the drag force model as discussed next.

Drag Force Model

Consider next a small element of the fly line in the front of the loop as shown in the next figure. This element subtends the infinitesimal angle $d\theta$ and has the infinitesimal length $Rd\theta$. The element is subjected to a component of drag that is tangent to the loop as denoted by the infinitesimal force $d\vec{D}_t$ in the figure. This drag component is referred to as *skin friction*. Skin friction acts upon the full surface area of the cylindrical element as given by $dA = \pi d_l R d\theta$. In addition, the element is subjected to a component of drag that is normal (perpendicular) to the loop as denoted by the infinitesimal force $d\vec{D}_n$ in the figure. This drag component is referred to as *form drag.* The form drag is proportional to the projected area of the element normal to loop as given by $dA = d_l R d\theta$.

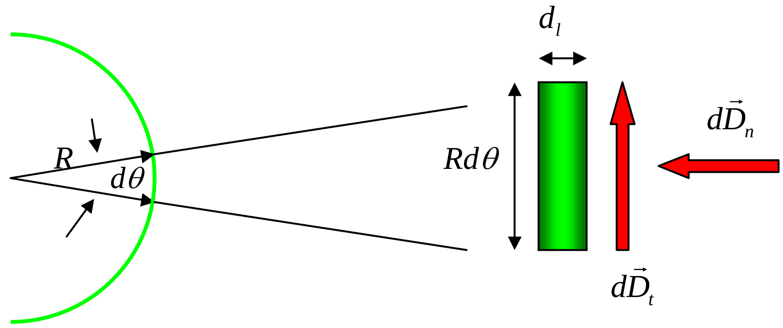

Figure 4. The normal drag component $d\vec{D}_n$ (form drag) and the tangential drag component $d\vec{D}_t$ (skin friction) acting on an infinitesimal element of fly line in the front of the loop.

We now introduce a suitable model for the skin friction and form drag. To this end, reconsider the velocity vector $\vec{v}_p$ shown in Fig. 2 and resolve this vector into components tangential and normal to the loop as indicated by the unit vectors $(\hat{t},\hat{n})$ shown in Fig. 5 below.

Figure 5. The velocity at point *p* is resolved into components along the local tangential and normal $(\hat{t},\hat{n})$.

Notice that

$$\hat{t} = \cos\theta\hat{i} - \sin\theta\hat{j} \tag{1.2}$$

$$\hat{n} = \sin\theta\hat{i} + \cos\theta\hat{j} \tag{1.3}$$

We can use Eqs. (1.2) and (1.3) together with Eq. (1.1) to compute the components of $\vec{v}_p$ along the tangential and normal as follows

tangential component

$$v_t = \vec{v}_p \bullet \hat{t} = \frac{v_o}{2}(1+\cos\theta) \tag{1.4}$$

normal component

$$v_n = \vec{v}_p \bullet \hat{n} = \frac{v_o}{2}\sin\theta \tag{1.5}$$

The infinitesimal skin friction and form drag are

skin friction

$$d\vec{D}_t = -\frac{1}{2}\rho_a \pi d_l R C_{dt} v_t^2 \hat{t} d\theta \tag{1.6}$$

form drag

$$d\vec{D}_n = -\frac{1}{2}\rho_a d_l R C_{dn} v_n^2 \hat{n} d\theta \tag{1.7}$$

in which C_{dt} and C_{dn} are the tangential drag (skin friction) coefficient and the normal drag (form drag) coefficient, respectively. Substituting the expressions for v_t and v_n from (1.4) and (1.5) into (1.6) and (1.7) provides us with a complete description of the infinitesimal skin friction and form drag acting on an infinitesimal element of fly line at any position θ on the loop front. We will now integrate (1.6) and (1.7) from $\theta = 0$ to $\theta = \pi$ in arriving at the net skin friction and form drag acting on the entire loop front. In addition, we will resolve these net drag forces into horizontal and vertical components. We begin with the vertical component.

Vertical Component of Drag on Loop Front, D_y

The vertical component of the net drag is computed from

$$D_y = \int_0^{\pi}[d\vec{D}_t + d\vec{D}_n]\bullet \hat{j} = \int_0^{\pi}[-\frac{1}{2}\rho_a \pi d_l R C_{dt} v_t^2 \hat{t}\bullet\hat{j} - \frac{1}{2}\rho_a d_l R C_{dn} v_n^2 \hat{n}\bullet\hat{j}]d\theta$$

Making use of Eqns. (1.2-1.5), this expands to

$$D_y = -\frac{1}{8}\rho_a d_l R v_o^2 \int_0^{\pi}[-\pi C_{dt}(1+\cos\theta)^2\sin\theta + C_{dn}\sin^2\theta\cos\theta]d\theta$$

The first term in the integrand captures the contribution of skin friction while the second term captures the contribution of form drag. The integral of the second term vanishes and therefore form drag makes no contribution to the net drag force in the vertical direction.[1] Performing the integration results in the final result for the vertical component of drag on the entire loop front.

$$D_y = \frac{\pi}{3}\rho_a d_l R C_{dt} v_o^2 \tag{1.8}$$

This upwards drag component opposes the weight of the loop given by

$$W = \frac{\pi^2}{4}\rho_l d_l^2 R g \tag{1.9}$$

Applying Newton's law in the vertical direction $(W - D_y = ma_y)$ allows one to compute the vertical acceleration of the loop

$$a_y = g - (\frac{4\rho_a}{3\rho_l}\frac{C_{dt}}{d_l})v_o^2 \tag{1.10}$$

where positive acceleration is taken as downwards.

We can use the simple result above for a_y to make several observations about how the loop drag opposes gravity.

1. Skin friction reduces the downwards acceleration of the loop.
2. This reduction grows with the square of the speed of the upper leg. Thus, a loop traveling twice as fast generates four times the "lift" due to skin friction.
3. This reduction is inversely proportional to the fly line diameter and the fly line density. Thus, "thin" lines of lesser density will generate more "lift" due to skin friction.
4. This reduction is proportional to the drag coefficient for skin friction. Thus, increasing this drag coefficient will generate more "lift" due to skin friction.
5. This reduction is independent of loop radius. Note however that larger loops will decelerate more rapidly leading to smaller v_o and therefore the dependence on loop diameter is really implicit through the speed v_o. The point is that smaller loops are generally launched with higher speeds and therefore they achieve greater initial "lift" due to skin friction.

Illustrative Examples

We will consider several examples that illustrate how the vertical acceleration of the loop as given by Eq. (1.10) varies with fly line properties and loop speed. We begin by studying how the skin friction drag coefficient C_{dt} affects a_y and as the loop slows down. Consider a typical floating fly line with diameter $d_l = 1.5mm$ and with density $\rho_l = 0.85 g/cm^3$. Let's assume a value of

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[1] This conclusion does not hold if the loop is allowed to fall under gravity. For a falling loop, form drag quickly develops and would indeed contribute a component in the vertically up direction acting opposite gravity.

$\rho_a = .00129 g/cm^3$ for the density of air and study how the vertical acceleration varies over a wide range of speed v_o for the upper segment. Results are shown in the figure below.

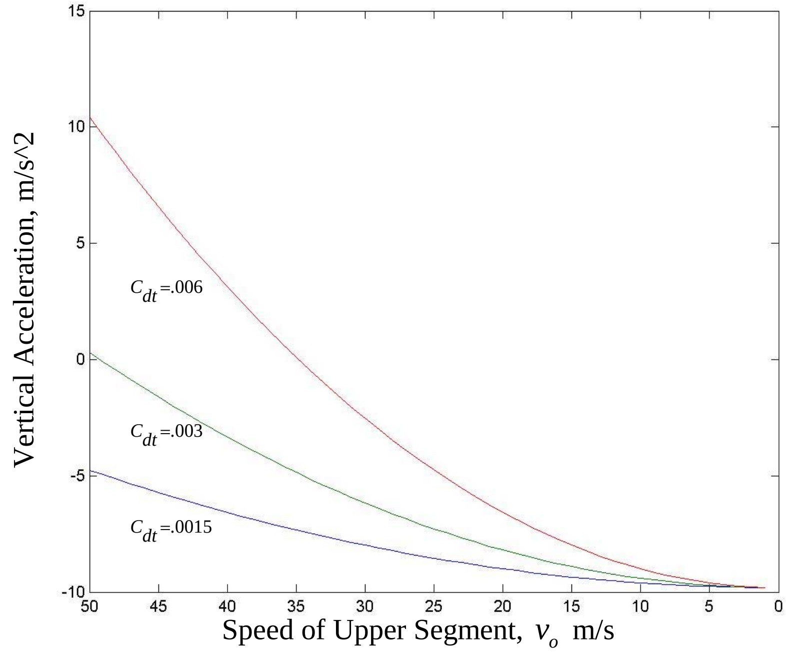

Figure 6. The vertical acceleration of the loop front as the cast unfolds from high speed to low speed. Results are shown for an example floating line for three different skin friction drag coefficients.

The speed v_o decreases as the cast unfolds and the resulting vertical acceleration decreases as well. Short length casting would likely involve speeds less than 25 m/s while distance casting may involve launch speeds that are two or three times higher. Thus, the speed range shown is reasonable. In all cases, the vertical acceleration converges to –g (-9.81 m/s) as the speed reaches zero since the drag forces vanish at zero speed. There is a significant dependence on the skin drag coefficient C_{dt} and while the value of this quantity is not well known, values in the range of .0015 and slightly higher have been suggested. The results show that at higher speeds, larger values of skin friction can “cancel” (or even overpower) gravity for a short period of time. This might help explain why tight loops, which are also necessarily launched at higher speed, might actually stay “aloft” longer even when launched purely in the horizontal direction.

Consider next how these results are affected by changing the fly line diameter. The results shown below contrast a line with the original diameter ($d_l = 1.5mm$) with a line having twice this diameter and a line having one half this diameter.

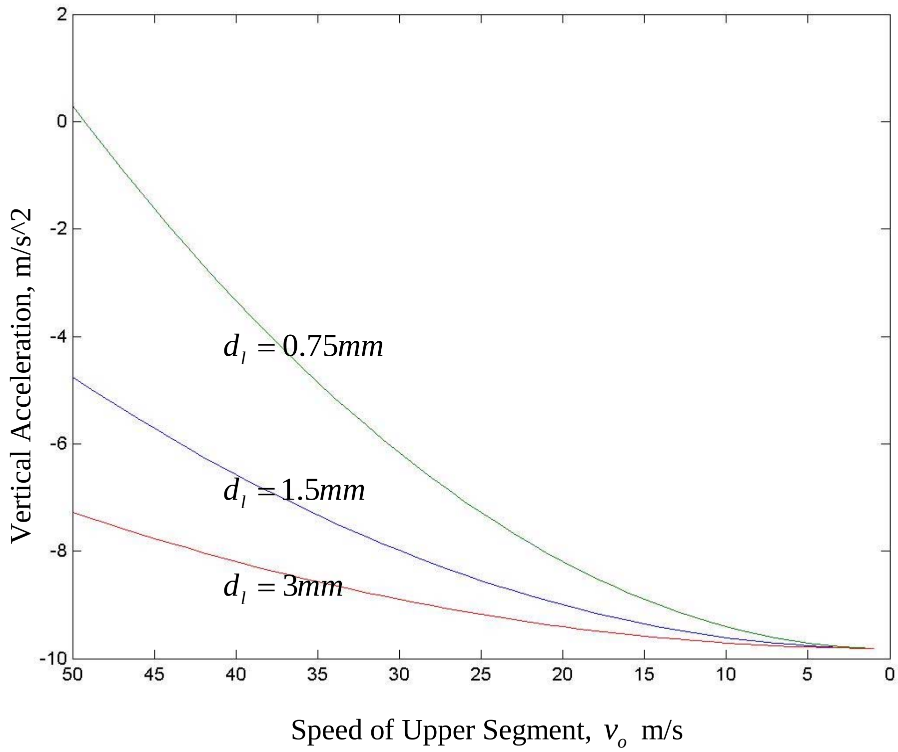

Figure 7. The vertical acceleration of the loop front as the cast unfolds from high speed to low speed. Results are shown for an example floating line for three different line diameters. These results can also be used to predict the effect of line density as explained below.

Clearly, there is also strong dependence on the fly line diameter as expected from Eq. (10) with smaller diameter lines experiencing less downwards acceleration for equal speed. This result expresses the simple fact that the fly line weight scales with the *square* of the fly line diameter, whereas the vertical drag force scales *linearly* with the fly line diameter. Thus, smaller diameter lines have a higher vertical drag to weight ratio (i.e., D_y / W varies as $1/d_l$).

Changing the fly line density leads to the very same conclusions as changing the fly line diameter above, namely the vertical drag to weight ratio is inversely proportional to the fly line density. (i.e., D_y / W varies as $1/\rho_l$). Thus, in the figure above, the curve for $d_l = 3mm$ is the same result as for a line with half the diameter ($d_l = 1.5mm$) but twice the density ($\rho_l = 1.7 g/cm^3$) resulting in a sinking line.

An important point to keep in mind is that the loop in this model is assumed to be moving horizontally and never vertically. The vertical component of drag that results from skin friction clearly opposes gravity and it will in general be smaller that gravity. Thus, the loop will ultimately accelerate downwards. As it does, then *form drag* will develop and contribute substantially to the drag in the vertical direction. This too will slow the fall of the loop.

Horizontal Component of Drag on Loop Front, D_x

The horizontal component of the net drag is computed from

$$D_x = \int_0^{\pi} [d\vec{D}_t + d\vec{D}_n] \bullet \hat{i} = \int_0^{\pi} [-\frac{1}{2}\rho_a \pi d_l R C_{dt} v_t^2 \hat{t} \bullet \hat{i} - \frac{1}{2}\rho_a d_l R C_{dn} v_n^2 \hat{n} \bullet \hat{i}] d\theta$$

Making use of Eqns. (1.2-1.5), this expands to

$$D_x = -\frac{1}{8}\rho_a d_l R v_o^2 \int_0^{\pi} [\pi C_{dt}(1+\cos\theta)^2 \cos\theta + C_{dn}\sin^2\theta \sin\theta] d\theta$$

The first term in the integrand captures the contribution of skin friction while the second term captures the contribution of form drag. The integral of the first term vanishes and therefore skin friction makes no contribution to the net drag force in the horizontal direction.[2] Performing the integration results in the final result for the horizontal component of drag on the entire loop front.

$$D_x = -\frac{1}{6}\rho_a d_l R C_{dn} v_o^2 \qquad (1.11)$$

This leftwards drag component decelerates the loop.

[2] This conclusion does not hold if the loop is allowed to fall under gravity. For a falling loop, skin friction would contribute to the net horizontal drag, albeit in a very modest way.